The dead-water phenomenon

A nonlinear approach

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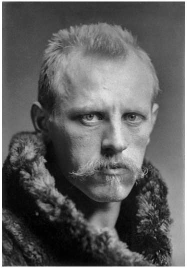
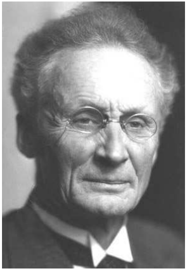
May 17, 2011

Discovery of the dead water phenomenon



Fridtjof Nansen, 1861–1930

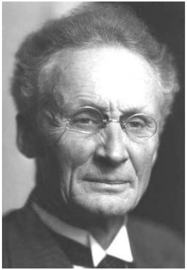
" When caught in dead water, **'Fram' appeared to be held back, as if by some mysterious force** (...) 'Fram' was capable of 6 to 7 knots. When in dead water she was unable to make 1.5 knots. We made loops in our course, turned sometimes right around, tried all sorts of antics to get clear of it, but to very little purpose. "



Vilhelm Bjerknes, 1862–1951

" I remarked that in the case of a layer of fresh water resting on the top of salt water, a ship will (...) generate invisible waves in the salt-water fresh water boundary below ; **I suggested that the great resistance experienced by the ship was due to the work done in generating these [internal] waves**."

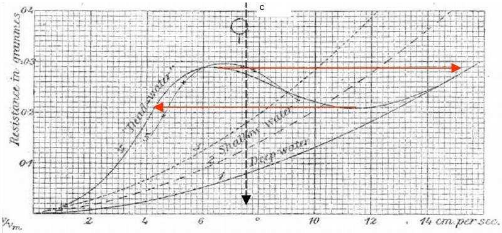
Discovery of the dead water phenomenon



Vilhelm Bjerknes, 1862–1951

“ **I suggested that the great resistance experienced by the ship was due to the work done in generating these [internal] waves**. (...) In December 1899 I consequently suggested a pupil of mine, Dr. V. Walfrid Ekman (...) that he should do some simple preliminary experiments.”

Vagn Walfrid Ekman, 1874–1954



Experiment of [Vasseur,Mercier,Dauxois (2008)]

Mathematical modelization

All existing models [1] are based on linear boundary conditions.

1. The governing equations
    - Our framework
    - Dirichlet-Neumann operators
    - The full Euler system
2. Several (coupled) asymptotic models
    - Expansion of the Dirichlet-Neumann operators
    - A fully nonlinear model (Green-Naghdi-type)
    - Weakly nonlinear models (Boussinesq-type)
3. An uncoupled model
    - The fKdV approximation
    - Rigorous justification
    - A consequence

1. **[Ekman (1904), Hudimac (1961), Sabunçu (1961), Price,Wang,Baar (1989), Miloh,Tulin,Zilman (1993), Nguyen,Yeung (1997), Motygin,Kuznetsov (1997), Ten,Kashiwagi (2004), Lu,Chen (2009)]**

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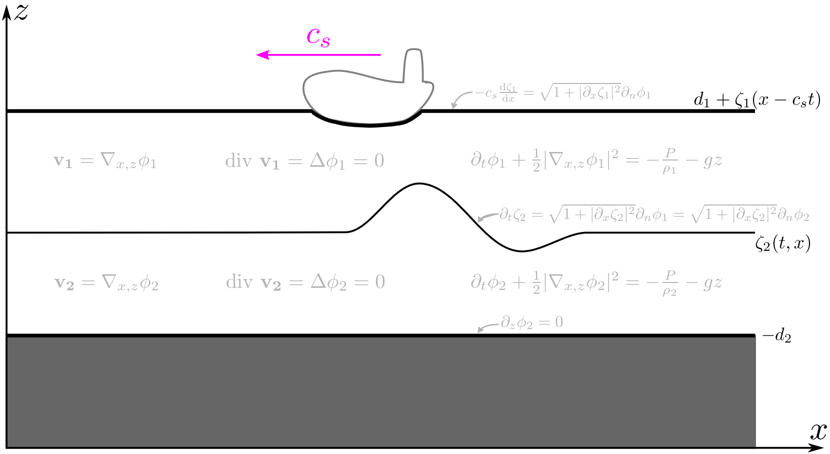
We follow a strategy similar to

- One layer case : **[Bona,Chen,Saut (2002,2004), Bona,Colin,Lannes (2005), Alvarez-Samaniego,Lannes (2008)]**
- Bi-fluidic case : **[Bona,Lannes,Saut (2008), VD (2010,2011)]**

Assumptions of our framework



- Dimension $d = 1$, flat bottom, fixed surface $\zeta_1 \equiv \zeta_1(x - c_s t)$.
- Irrotational, incompressible, inviscid, immiscible fluids.
- Fluids at rest at infinity, no surface tension.

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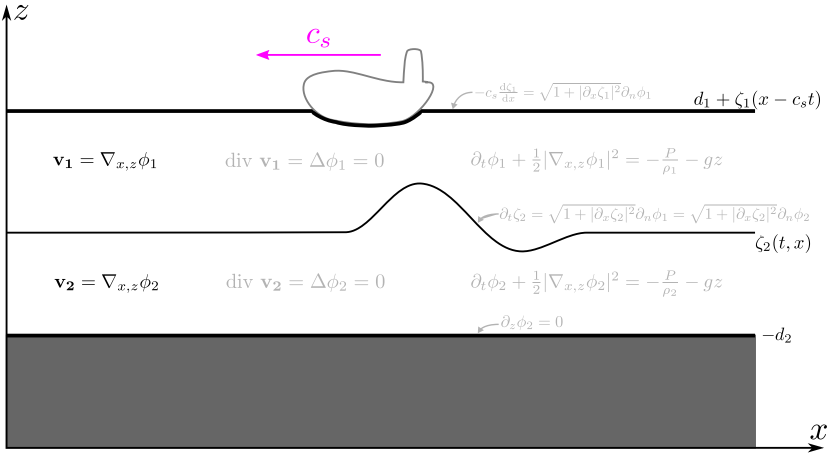

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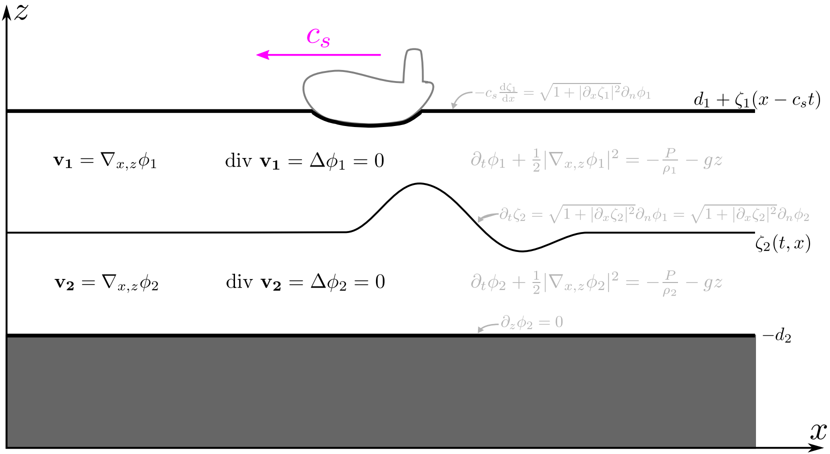

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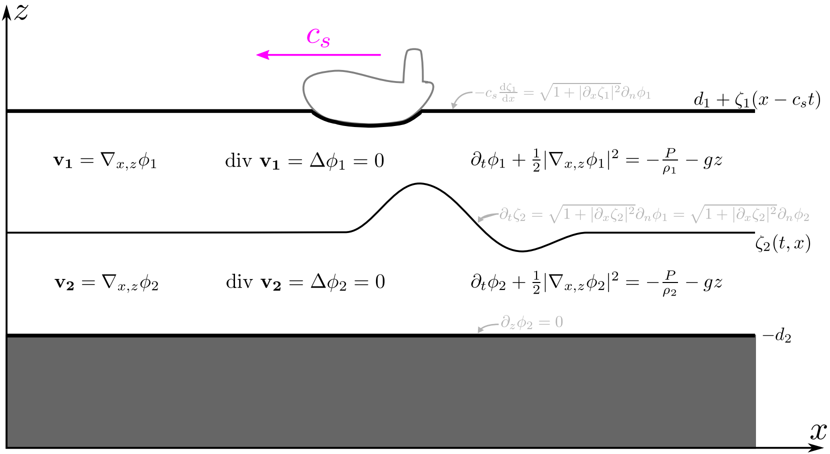

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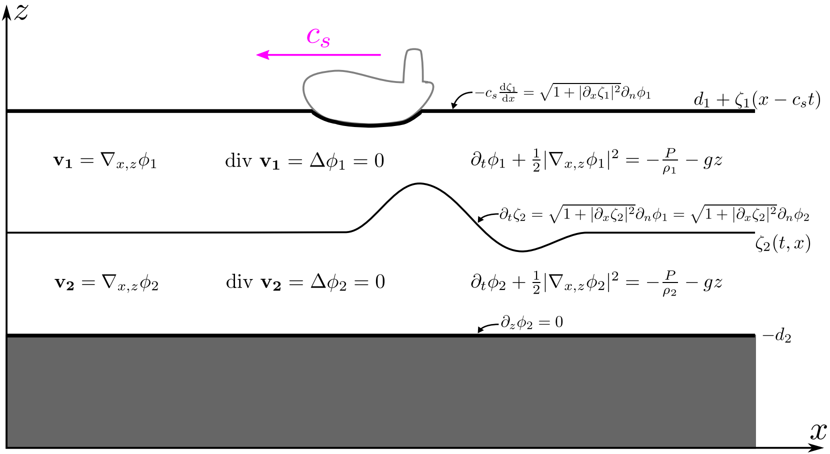

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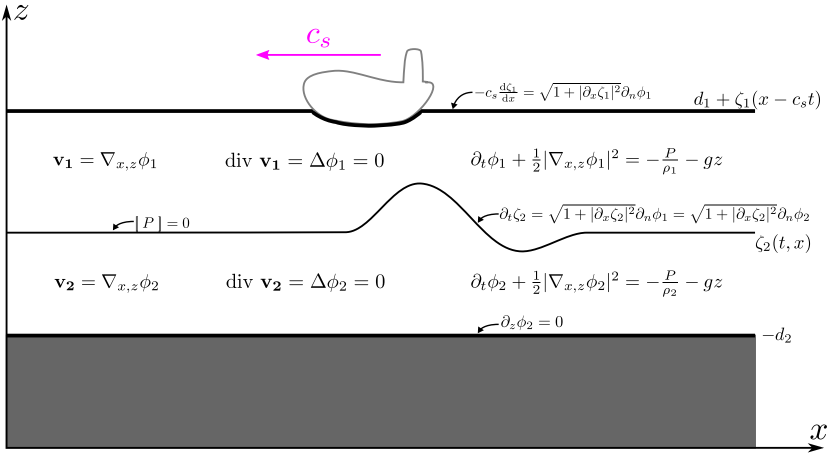

- Dimension $d = 1$, flat bottom, fixed surface $\zeta_1 \equiv \zeta_1(x - c_s t)$.
- Irrotational, incompressible, inviscid, immiscible fluids.
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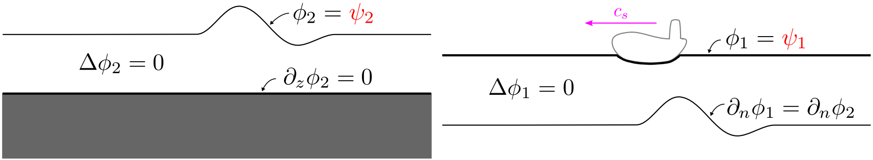
Dirichlet-Neumann operators

The equations can be reduced to evolution equations located on the surface and on the interface thanks to the following operators



$$\begin{cases} \Delta_{x,z}\phi_2 = 0 & \text{in } \{-d_2 < z < \zeta_2\}, \\ \phi_2 = \psi_2 & \text{on } \{z = \zeta_2\}, \\ \partial_z\phi_2 = 0 & \text{on } \{z = -d_2\}, \end{cases} \qquad \begin{cases} \Delta_{x,z}\phi_1 = 0 & \text{in } \{\zeta_2 < z < d_1 + \zeta_1\}, \\ \phi_1 = \psi_1 & \text{on } \{z = d_1 + \zeta_1\}, \\ \partial_{n_2}\phi_1 = \partial_{n_2}\phi_2 & \text{on } \{z = \zeta_2\}. \end{cases}$$

$$\downarrow \qquad\qquad \downarrow$$

$$\phi_2 \qquad\qquad \phi_1$$

Therefore, the system is entirely defined by

$$\zeta_1 \quad ; \quad \zeta_2 \quad ; \quad \psi_1 \equiv \phi_{1|\text{surface}} \quad ; \quad \psi_2 \equiv \phi_{2|\text{interface}}.$$

Dirichlet-Neumann operators

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Dirichlet-Neumann operators

The following operators are well-defined :

$$G_2\psi_2 \equiv \sqrt{1+|\partial_x\zeta_2|^2}\partial_n\phi_{2|\text{interface}},$$
$$G_1(\psi_1,\psi_2) \equiv \sqrt{1+|\partial_x\zeta_1|^2}\partial_n\phi_{1|\text{surface}},$$
$$H(\psi_1,\psi_2) \equiv \partial_x\Big(\phi_{1|\text{interface}}\Big).$$

Therefore, the system is entirely defined by

$$\zeta_1 \quad ; \quad \zeta_2 \quad ; \quad \psi_1 \equiv \phi_{1|\text{surface}} \quad ; \quad \psi_2 \equiv \phi_{2|\text{interface}}.$$

The full Euler system

The dimensionless full Euler system

$$\begin{cases} -c_s\partial_x\zeta_1 \ - \ G_1(\psi_1,\psi_2) \ = \ 0, \\ \partial_t\zeta_2 \ - \ G_2\psi_2 \ = \ 0, \\ \partial_t\Big(\rho_2\partial_x\psi_2 - \rho_1 H(\psi_1,\psi_2)\Big) + g(\rho_2-\rho_1)\partial_x\zeta_2 \\ \qquad +\frac{1}{2}\partial_x\Big(\rho_2|\partial_x\psi_2|^2 - \rho_1|H(\psi_1,\psi_2)|^2\Big) \ = \ \partial_x\mathcal{N}_2, \end{cases} \qquad (\Sigma)$$

Using that ζ_1 is some fixed data, the system reduces to two evolution equations for (ζ_2, v), with v the shear velocity defined by

$$v \ \equiv \ \partial_x\Big(\big(\rho_2\phi_2-\rho_1\phi_1\big)_{|z=\varepsilon\zeta_2}\Big) \ = \ \rho_2\partial_x\psi_2-\rho_1 H(\psi_1,\psi_2).$$

Solutions of this system are *exact* solutions of our problem. We construct asymptotic models, and therefore look for approximate solutions.

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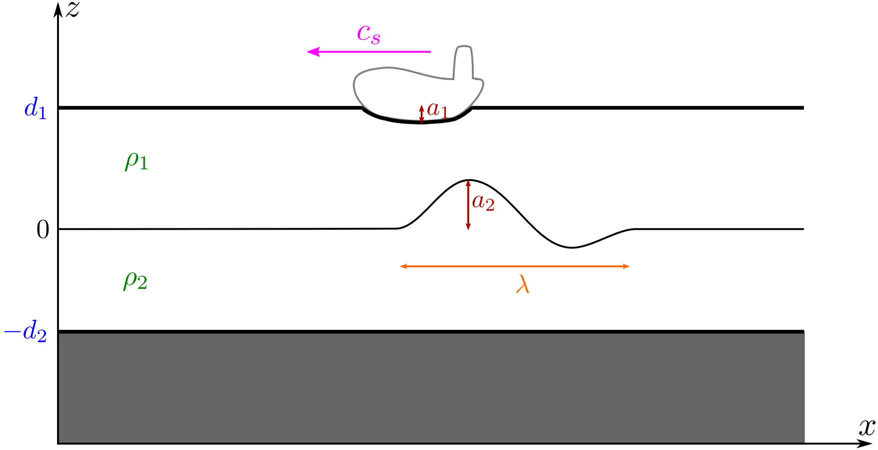
Nondimensionalizing the system



$$\epsilon_1 \equiv \frac{a_1}{d_1}, \quad \epsilon_2 \equiv \frac{a_2}{d_1}, \quad \mu \equiv \frac{{d_1}^2}{\lambda^2}, \quad \gamma \equiv \frac{\rho_1}{\rho_2}, \quad \delta \equiv \frac{d_1}{d_2}, \quad \mathrm{Fr} = \frac{c_s}{c_0}.$$

$\mu \ll 1, \qquad \epsilon_1/\epsilon_2 = \mathcal{O}(\mu), \qquad \epsilon_2 = \mathcal{O}(1)$ or $\mathcal{O}(\mu).$

Expansion of the Dirichlet-Neumann operators

The dimensionless full Euler system

$$\begin{cases} -\frac{\epsilon_1}{\epsilon_2}\,\mathrm{Fr}\,\partial_x\zeta_1 \;-\; \frac{1}{\mu}G_1(\psi_1,\psi_2) \;=\; 0, \\ \partial_t\zeta_2 \;-\; \frac{1}{\mu}G_2\psi_2 \;=\; 0, \\ \partial_t\Big(\partial_x\psi_2 - \gamma H(\psi_1,\psi_2)\Big) + (\gamma+\delta)\partial_x\zeta_2 + \frac{\epsilon_2}{2}\partial_x\Big(|\partial_x\psi_2|^2 - \gamma|H(\psi_1,\psi_2)|^2\Big) \\ \qquad\qquad = \;\mu\epsilon_2\partial_x\mathcal{N}_2, \end{cases} \qquad (\tilde\Sigma)$$

Proposition

Let $s>1$, $\zeta_1,\ \zeta_2,\ \psi_1,\ \psi_2\ \in\ H^{s+t}(\mathbb{R})$. Then one has

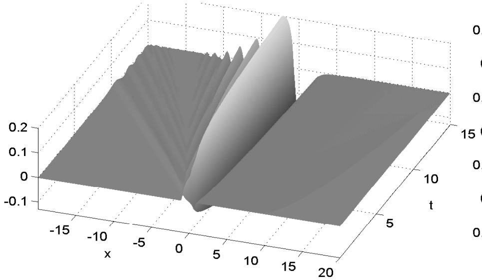
$$\Big|G_2\psi_2 \;-\; \mu\mathcal{G}_{2,1} \;-\; \mu^2\mathcal{G}_{2,2}\Big|_{H^s} \;\leq\; \mu^3 C$$

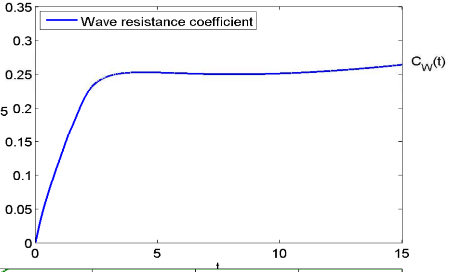
$$\Big|G_1(\psi_1,\psi_2) \;-\; \mu\mathcal{G}_{1,1} \;-\; \mu^2\mathcal{G}_{1,2}\Big|_{H^s} \;\leq\; \mu^3 C,$$

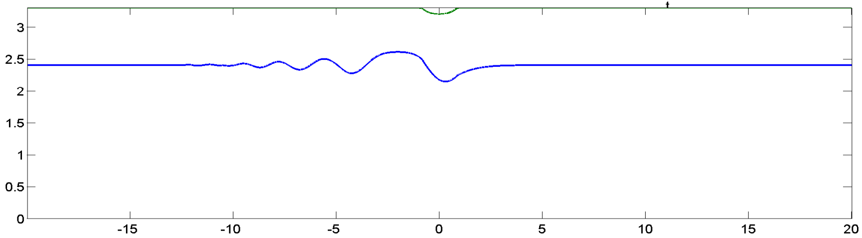
$$\Big|H(\psi_1,\psi_2) \;-\; \partial_x\psi_1 \;-\; \mu\mathcal{H}_1\Big|_{H^s} \;\leq\; \mu^2 C,$$

A strongly nonlinear model :

$$\epsilon_1/\epsilon_2,\ 1-\gamma\ =\ \mathcal{O}(\mu),\ \mu\ \ll\ 1.$$







A weakly nonlinear model :

$$\epsilon_1/\epsilon_2 = \epsilon_2 = \mu \equiv \varepsilon \ll 1.$$

A Boussinesq/Boussinesq model

$$\begin{cases} \partial_t\zeta_2 + \frac{1}{\delta+\gamma}\partial_x v + \varepsilon\frac{\delta^2-\gamma}{(\gamma+\delta)^2}\partial_x(\zeta_2 v) + \varepsilon\frac{1+\gamma\delta}{3\delta(\delta+\gamma)^2}\partial_x^3 v = -\varepsilon\frac{\mathrm{Fr}\,\gamma}{\delta+\gamma}\partial_x\zeta_1, \\ \partial_t v + (\gamma+\delta)\partial_x\zeta_2 + \frac{\varepsilon}{2}\frac{\delta^2-\gamma}{(\gamma+\delta)^2}\partial_x\left(|v|^2\right) = 0. \end{cases}$$

$$\hookrightarrow \partial_t U + A_0\partial_x U + \varepsilon\left(A_1(U)\partial_x U + A_2\partial_x^3 U \right) = \varepsilon\partial_x F(x-\mathrm{Fr}\,t) \qquad (\mathcal{M}_B)$$

with $U = (\zeta_2, v)$, $F : \mathbb{R} \to \mathbb{R}^2$.

Consistency

The full Euler system is consistent with the Boussinesq/Boussinesq model, with precision $\mathcal{O}(\varepsilon^2)$.

Open questions :

- Well-posedness of any Boussinesq/Boussinesq system ?
- Convergence of solutions towards solutions of the full Euler system ?

Symmetrization

We have a system of the form

$$\partial_t U \ + \ A_0 \partial_x U \ + \ \varepsilon \left(\ A_1(U)\partial_x U \ + \ A_2 \partial_x^3 U \ \right) \ = \ \varepsilon \partial_x F(x - \mathrm{Fr}\, t).$$

Multiply by adapted $S \ \equiv \ S_0 \ + \ \varepsilon S_1(U) \ - \ \varepsilon S_2 \partial_x^2$, and withdraw $\mathcal{O}(\varepsilon^2)$ terms. One obtains a perfectly symmetric model of the form :

The symmetric Boussinesq/Boussinesq model

$$\Big(S_0 + \varepsilon\big(S_1(U) - S_2\partial_x^2\big)\Big)\partial_t U + \Big(\Sigma_0 + \varepsilon\big(\Sigma_1(U) - \Sigma_2\partial_x^2\big)\Big)\partial_x U = \varepsilon \partial_x G, \qquad (\mathcal{S}_B)$$

with the following properties :

- Matrices S_0, S_2, Σ_0, $\Sigma_2 \in \mathcal{M}_2(\mathbb{R})$ are symmetric.
- $S_1(\cdot)$ and $\Sigma_1(\cdot)$ are linear mappings, with values in $\mathcal{M}_2(\mathbb{R})$, and for all $U \in \mathbb{R}^2$, $S_1(U)$ and $\Sigma_1(U)$ are symmetric.
- S_0 et S_2 are definite positive.

Rigorous justification of the model

Consistency

The full Euler system is consistent with the symmetric Boussinesq/Boussinesq model $(\mathcal{S}_B)$, with precision $\mathcal{O}(\varepsilon^2)$.

Well posedness

The symmetric system is well-posed in H^{s+1} $(s > 3/2)$ over times of order $\mathcal{O}(1/\varepsilon)$. Moreover, one has the estimate

$$\left(\left| U(t) \right|_{H^s}^2 + \varepsilon \left| U(t) \right|_{H^{s+1}}^2 \right)^{1/2} = \left| U(t) \right|_{H^{s+1}_\varepsilon} \leq C_0 (e^{C_1 \varepsilon t} - 1) \ .$$

Convergence

The difference between any solution U of the full Euler system (Σ), and the solution U_B of the symmetric Boussinesq/Boussinesq model $(\mathcal{S}_B)$ with same initial data, satisfies

$$\forall t \in [0, T/\varepsilon], \qquad \left| U - U_B \right|_{L^\infty([0,t];H^{s+1}_\varepsilon)} \leq \ \varepsilon C_2 (e^{C_3 \varepsilon t} - 1).$$

▸ Numerical simulation

The WKB expansion

We seek an approximate solution of system $(\mathcal{S}_B)$:

$$\Big(S_0+\varepsilon\big(S_1(U)-S_2\partial_x^2\big)\Big)\partial_t U+\Big(\Sigma_0+\varepsilon\big(\Sigma_1(U)-\Sigma_2\partial_x^2\big)\Big)\partial_x U=\varepsilon\partial_x G$$

At first order : $(S_0\partial_t+\Sigma_0\partial_x)U_0 \ = \ 0.$

There exists a basis $\mathbf{e}_i$ diagonalizing S_0 and Σ_0 :

$$\Longrightarrow U_0=\sum u_i\mathbf{e}_i \quad \text{, with} \quad \partial_t u_i(t,x)+c_i\partial_x u_i \ = \ 0.$$

At next order : We seek an approximation of the form

$$U_{\rm app}(t,x) \ \equiv \ \sum u_i(\varepsilon t,t,x)\mathbf{e}_i \ + \ \varepsilon U_1(\varepsilon t,t,x). \qquad \text{(WKB)}$$

$$\partial_t u_i(t,x)+c_i\partial_x u_i \ = \ 0, \qquad (1)$$

$$\partial_\tau u_i \ + \ \lambda_i u_i\partial_{x_i}u_i \ + \ \mu_i\partial_{x_i}^3 u_i \ = \ \partial_x g_i, \qquad (2)$$

$$(\partial_t+c_i\partial_x)\mathbf{e}_i\cdot U_1 \ + \sum_{(j,k)\neq(i,i)}\alpha_{ijk}u_k\partial_x u_j \ + \ \sum_{j\neq i}\beta_{ij}\partial_x^3 u_j \ = \ 0. \qquad (3)$$

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The fKdV Approximation

Definition

We define then the fKdV approximation as $U_{fKdV} = \sum u_i \mathbf{e}_i$, where u_i satisfies

$$\begin{cases} \partial_t u_i + c_i \partial_x u_i + \varepsilon \lambda_i u_i \partial_x u_i + \varepsilon \mu_i \partial_x^3 u_i = \varepsilon \partial_x g_i, \\ u_{i|t=0} = u_i^0, \end{cases}$$

In our case, this yields

The fKdV approximation

$U_{\text{fKdV}} = (\zeta, v) = \big(\zeta_+ + \zeta_- , (\gamma+\delta)(\zeta_+ - \zeta_-) \big)$, with

$$\partial_t \zeta_\pm + (-\operatorname{Fr} \pm 1)\partial_x \zeta_\pm \pm \varepsilon \frac{3}{2} \frac{\delta^2-\gamma}{\gamma+\delta} \zeta_\pm \partial_x \zeta_\pm \pm \varepsilon \frac{1}{6} \frac{1+\gamma\delta}{\delta(\gamma+\delta)} \partial_x^3 \zeta_\pm = -\varepsilon \operatorname{Fr} \gamma \frac{d\zeta_1}{dx}. \tag{fKdV}$$

Rigorous justification (1/2)

Well-posedness

If $U^0 \in H^{s+2}$, then there exists a unique strong solution $U_0(\tau, t, x)$, uniformly bounded in $C^1([0,T] \times \mathbb{R}; H^{s+2})$.
The residual U_1 is explicit, and $U_1 \in C^1([0,T] \times \mathbb{R}; H^s)$.

Secular growth of the residual

$$\forall (\tau, t) \in [0,T] \times \mathbb{R}, \quad \big| U_1(\tau, t, \cdot) \big|_{H^s} \leq C_0 \sqrt{t}.$$

Moreover, if $(1+x^2)U_0 \in H^{s+2}$, then one has the uniform estimate

$$\big| U_1(\tau, t, \cdot) \big|_{H^s} \leq C_0,$$

Consistency

$U_0(\varepsilon t, t, x) \ + \varepsilon U_1(\varepsilon t, t, x)$ satisfies the symmetric Boussinesq/Boussinesq model $(\mathcal{S}_B)$, with precision $\mathcal{O}(\varepsilon^{3/2})$ (and $\mathcal{O}(\varepsilon^2)$ if $(1+x^2)U^0 \in H^{s+2}$).
$\Longrightarrow$ convergence towards the solution of $(\mathcal{S}_B)$.

Rigorous justification (2/2)

The fKdV approximation

$U_{\mathrm{fKdV}} = (\zeta, v) \ = \ \big(\ \zeta_+ + \zeta_- \ , \ (\gamma+\delta)(\zeta_+ - \zeta_-)\ \big)$, with

$$\partial_t \zeta_\pm \ + \ (-\,\mathrm{Fr} \pm 1)\partial_x \zeta_\pm \ \pm \ \varepsilon \frac{3}{2}\frac{\delta^2-\gamma}{\gamma+\delta}\zeta_\pm \partial_x \zeta_\pm \ \pm \ \varepsilon \frac{1}{6}\frac{1+\gamma\delta}{\delta(\gamma+\delta)}\partial_x^3 \zeta_\pm \ = \ -\varepsilon\,\mathrm{Fr}\,\gamma \frac{\mathrm{d}\zeta_1}{\mathrm{d}x}. \tag{fKdV}$$

Convergence towards solutions of the full Euler system

The difference between any solution U of the full Euler system (Σ), and $U_{\mathrm{fKdV}} \equiv \ \big(\ \zeta_+ + \zeta_- \ , \ (\gamma+\delta)(\zeta_+ - \zeta_-)\ \big)$ satisfies

$$\big| U - U_{\mathrm{fKdV}} \big|_{L^\infty([0,t];H^{s+1}_\varepsilon)} \leq \varepsilon \sqrt{t}\, C_0.$$

Moreover, if $(1+x^2)U\big|_{t=0} \in H^{s+5}$, then one has the uniform estimate

$$\big| U - U_{\mathrm{fKdV}} \big|_{L^\infty([0,T/\varepsilon];H^{s+1}_\varepsilon)} \leq \varepsilon C_0.$$

A simple application

Lemma

Let u be the solution of

$$\partial_t u \ + \ c\partial_x u \ + \ \varepsilon\lambda u\partial_x u \ + \ \varepsilon\nu\partial_x^3 u \ = \ \varepsilon\partial_x f(x - c_0 t),$$

with $u_{|t=0} \ = \ \varepsilon u^0 \in H^{s+3}$, $s > 3/2$.

There exists $T(|\frac{1}{c-c_0}|)$ and $C(|\frac{1}{c-c_0}|) > 0$ such that

$$\big| \ u \ \big|_{L^\infty([0,T/\varepsilon];H^s)} \ \leq \ C\varepsilon.$$

The solution of the transport equation $\partial_t v \ + \ c\partial_x v \ = \ \varepsilon\partial_x f(x - c_0 t)$ is

$$v \ = \ \varepsilon u^0(x - ct) \ + \ \frac{\varepsilon}{c_0 - c}\big(\ f(x - c_0 t) - f(x - ct) \ \big).$$

The result is obtained by comparison with this function.

As a consequence, the dead-water phenomenon will always be small if the velocity of the body is away from the critical velocity ($|\,\mathrm{Fr}\,| \ = \ 1$).

A simple application

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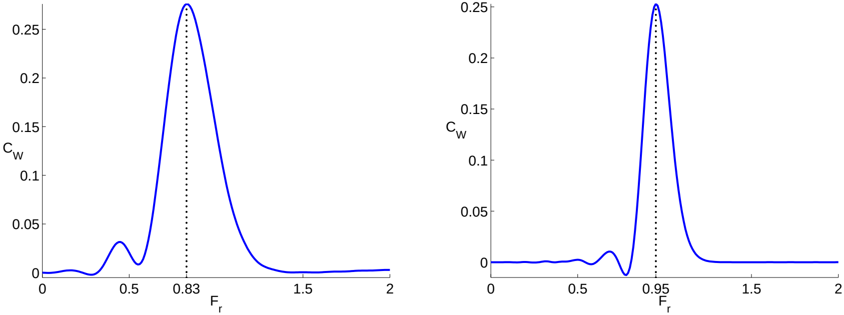

Wave resistance coefficient C_W at time $T = 10$, depending on the Froude number Fr, with $\delta = 5/12$ and $12/5$ ($\gamma = 0.9$, $\varepsilon = 0.1$).

Conclusion

We constructed <u>nonlinear</u> models that recover most of the key features of the dead water phenomenon :

- transverse internal waves are generated ;
- positive drag when an internal elevation wave is located at the stern ;
- the effect is strong only near critical Froude numbers ;
- the maximum peak of the drag is reached at slightly subcritical values.

We do NOT recover :

- divergent waves (requires horizontal dimension $d = 2$) ;
- hysteretic aspect (requires "constant-force" models).

Thanks for your attention !

References :

Mercier, Vasseur, Dauxois, *Resurrecting dead-water phenomenon.* Nonlinear Processes Geophys. (2011).

VD, *Asymptotic models for the generation of internal waves by a moving ship, and the dead-water phenomenon.* ArXiv preprint :1012.5892

The resistance coefficient

Definition (Wave resistance)

$$R_W \equiv \int_{\Gamma_{\text{ship}}} P\,(-\mathbf{e}_x \cdot \mathbf{n})\,dS = -\int_{\mathbb{R}} P_{|d_1+\zeta_1} \partial_x \zeta_1\,dx.$$

where Γ_{ship} is the exterior domain of the ship, P is the pressure, $\mathbf{e}_x$ is the horizontal unit vector and $\mathbf{n}$ the normal unit vector exterior to the ship.

As a solution of the Bernoulli equation, the pressure P satisfies

$$\frac{P(x,z)}{\rho_1} = -\partial_t \phi_1(x,z) - \frac{1}{2}|\nabla_{x,z}\phi_1(x,z)|^2 - gz.$$

Using change of variables, we deduce the dimensionless wave resistance coefficient C_W. If $\epsilon_2 = \mu = \epsilon_1/\epsilon_2 \equiv \varepsilon \ll 1$, the first order approximation is

$$C_W = \int_{\mathbb{R}} \zeta_1 \partial_x \zeta_2\,dx + \mathcal{O}(\varepsilon).$$

The "constant-force" hypothesis

When adjusting the velocity of the body at each time step

$$\text{Fr}((n+1)\Delta t) \equiv \text{Fr}(n\Delta t) - \Delta t \text{Cstt}_1(C_W(n\Delta t) - \text{Cstt}_2).$$